Group Delay and Dissipation Loss in Transmission-Line Filters*

SUMMARY

Basic formulas for the group delay and for the dissipation loss are derived for transmission-line filters. The optimum number of resonators for minimum dissipation loss is found.

INTRODUCTION

The purpose of this communication is to present the basic formulas for the midband group delay and dissipation loss for transmisssion-line filters in terms of transmission-line parameters. One would expect the group delay and dissipation loss at frequencies in the pass band to remain close to their midband values. Numerical solutions[1–3] have confirmed that the variation over the pass band region is usually not great.

The delay and dissipation loss are derived using a stepped-impedance filter prototype.[4] Quarter-wave transformers and half-wave filters are examples of stepped-impedance filters, and are shown in Figs. 1 and 2.

The relationship between group delay and dissipation loss will also be established. A simple formula will be derived for the optimum number of resonators for minimum dissipation loss when the stop-hand attenuation is specified.

GROUP DELAY

Consider a single-section quarter-wave transformer (Fig. 3). By direct multiplication of the transmission matrices (as for instance in Young[5]) it can be shown that the ratio of the transmitted wave amplitude to the incident wave amplitude is

$$\frac{a_3}{a_1} = \frac{T^2 e^{-j\theta}}{1 + \Gamma^2 e^{-2j\theta}}. \tag{1}$$

Denoting the phase change by ϕ,

$$\phi = \arg\left(\frac{a_3}{a_1}\right). \tag{2}$$

By differentiation, at center frequency

$$\left.\frac{d\phi}{d\theta}\right|_{\theta=\pi/2} = \frac{S^2+1}{2S} = U \tag{3}$$

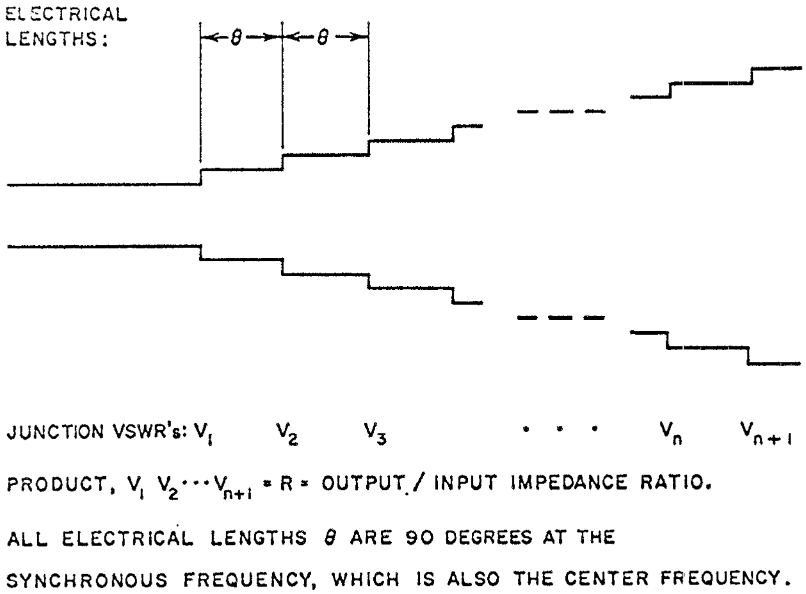

Fig. 1—Quarter-wave transformer as a prototype circuit.

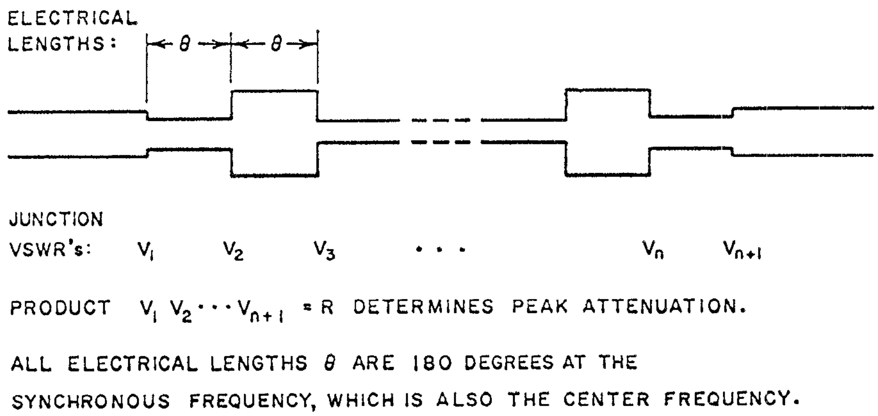

Fig. 2—Half-wave stepped filter as a prototype circuit.

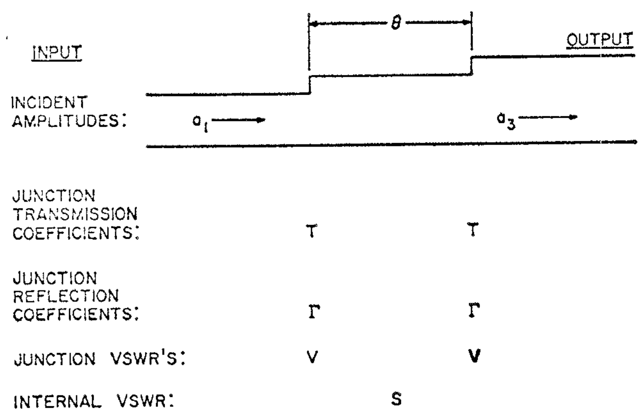

Fig. 3—Single-section quarter-wave transformer.

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[1] S. B. Cohn, "Phase-shift and time-delay response of microwave narrow-band filters," *The Microwave J.*, vol. 3, pp. 47–51; October, 1960.

[2] L. Young, "Suppression of Spurious Frequencies," Stanford Res. Inst., Menlo Park, Calif., Quart. Prog. Rept. 1, Project 4096, Contract No. AF 30(602)-2734; July, 1962.

[3] G. L. Matthaei, L. Young, and E. M. T. Jones, "Design of Microwave Filters, Impedance-Matching Networks, and Coupling Structures," Project 3527, Contract No. DA 36-039 SC-87398, vol. I, chs. 4, 6; January, 1963.

[4] L. Young, "Stepped-impedance transformers and filter prototypes," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 339–359; September, 1962.

[5] L. Young, "Analysis of a transmission wavemeter," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 436–439; July, 1960.

where U is defined[6] by

$$U = \frac{\text{Gross Power Flow}}{\text{Net Power Flow}} \tag{4}$$

at center frequency inside the transformer section (or later, filter cavity), and S is the internal VSWR at center frequency (Fig. 3). For an n-section transformer

$$\frac{d\phi}{d\theta} = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{S_i^2 + 1}{2S_i} \tag{5}$$

(which also holds for a half-wave filter), where S_i is the internal VSWR in the ith section or cavity.

The group delay t_d in the pass band is given very nearly by the radian frequency derivative of the phase,

$$t_d = \frac{d\phi}{d\omega} = \frac{1}{2\pi}\frac{d\phi}{df} = \frac{1}{2\pi}\frac{d\phi}{d\theta}\cdot\frac{d\theta}{df}. \tag{6}$$

(Some authors prefer to take this equation as the *definition* of group delay.) Hence, at center frequency f_0,

$$f_0(t_d)_0 = \frac{1}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \sum_{i=1}^{n} U_i \tag{7}$$

for the *quarter-wave transformer*, since

$$f_0 \frac{d\theta}{df} = \frac{\pi}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \tag{8}$$

where λ_g is the guide wavelength, and λ is the free-space wavelength. The suffix 0 denotes that the quantity is to be taken at center frequency f_0.

For the *half-wave filter* (7) becomes

$$f_0(t_d)_0 = \frac{1}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \sum_{i=1}^{n} U_i \tag{9}$$

while in the general case of a stepped-impedance filter all of whose section lengths l_i are multiples of a quarter wavelength, the group delay is given by

$$f_0(t_d)_0 = \sum_{i=1}^{n} \left(\frac{\lambda_{gi}}{\lambda}\right)_0^2 \left(\frac{l_i}{\lambda_{gi}}\right)_0 U_i. \tag{10}$$

In (7), (9), and (10), U_i is derived from

$$U_i = \frac{S_i^2 + 1}{2S_i} \tag{11}$$

where S_i is the internal VSWR seen inside the ith cavity at center frequency and is easily calculated for synchronously tuned[4] filters. (All the filters considered here are synchronously tuned filters.) Then the internal VSWR of any cavity is deduced from the internal VSWR of the neighboring cavity by forming its product or quotient with the VSWR of the step separating them. In any particular case it will be clear how to determine the S_i. It can be shown[3,6] that this leads to the following formula for S_i for synchronously tuned filters in general:

$$S_i = \left(\frac{V_{i+1}V_{i+3}V_{i+5}\cdots}{V_{i+2}V_{i+4}\cdots}\right)^{\pm 1} > 1 \tag{12}$$

with V_n and V_{n+1} terminating the two products in the numerator and the denomitator, or vice versa.

In the case of narrow-band filters (the case of interest here) a simpler equation can be shown[3,6] to hold:

$$V_i = S_i S_{i-1} \quad (i = 1, 2, \cdots, n+1) \tag{13}$$

Since the output is matched ($S_{n+1}=1$), this yields all the S_i when the V_i are given.

Dissipation Loss

It can be shown[3,6] that $(\Delta L_A)_0$, the increase in loss due to dissipation at center frequency, when small, is given by

$$(\Delta L_A)_0 = (1 - |\rho_0|^2) \sum_{i=1}^{n} \alpha_i l_i U_i \tag{14}$$

where ρ_0 is the input reflection coefficient, l_i are the resonator line lengths (all multiples of a quarter wavelength), U_i is defined by (4) or (11), and α_i is the attenuation per unit length. If the α_i are expressed in decibels per unit length, then $(\Delta L_A)_0$ is in decibels (similarly for nepers, etc.). If the filter is matched at band-center ($\rho_0=0$), and if further the filter is homogeneous (all λ_{gi} the same), and if all the α_i are the same, then from (10) and (14),

$$(\Delta L_A)_0 = (\alpha\lambda_g)(\lambda/\lambda_g)^2 f_0(t_d)_0. \tag{15}$$

Eq. (15) has been proved only for the center frequency; however, it may be expected to hold closely over the entire pass band region, for the following reasons: The dissipation loss, when small, is proportional to the stored energy, which for a reflectionless uniform transmission line would be proportional to the delay through the line. This may be expected to hold approximately for loaded lines and filters in their pass band. We may, therefore, drop the suffix 0 in (15) and write approximately

$$\Delta L_A \approx (\alpha\lambda_g)(\lambda/\lambda_g)^2 f_0 t_d \tag{16}$$

in the pass band region.

A formula for lumped-constant filters has been given by Cohn.[7] This can be shown[3,6] to be equivalent to (14) for narrow-band, selective filters.

Concerning the relationship between dissipation loss and group delay of lumped-constant filters, it can be shown in Young[6,8] that when all the resonators have the same value of unloaded Q, Q_u, then (16) reduces to

$$\Delta L_A = \frac{\pi}{Q_u} f_0 t_d \text{ nepers} = \frac{2.73}{Q_u} f_0 t_d \text{ decibels}. \tag{17}$$

Eq. (17) has also been derived for lumped-constant filters by Matthaei,[3] using an equation of Bode.[9]

Minimum Dissipation Loss and Optimum Number of Resonators

Consider the case of a filter matched at center frequency. Suppose that all the $\alpha_i l_i$ are equal; or we may suppose that each cavity has the same length and Q_u. Then find the filter with minimum center-frequency dissipation loss when R is given. In this case it has been shown[3,6] that the solution requires all the internal VSWRs, S_i to be equal; such a filter is called a periodic filter.[2,3] In that case

$$S_i = R^{1/2n} \tag{18}$$

and (14) for large R reduces to

$$(\Delta L_A)_{0,\min} \approx n\alpha l \left(\frac{R^{1/2n}}{2}\right). \tag{19}$$

This may be stated in words as follows:

"The minimum center-frequency dissipation loss of a matched filter, consisting of large discontinuities placed along a uniform transmission line, is the attenuation of the same length of line when the discontinuities are removed, multiplied by one half of the $(2n)$th root of the given R of the filter. This loss is attained only in the periodic filter."

The minimum band-center dissipation loss for a given R is obtained with a periodic filter and is given by Eq. (19). Differentiating with respect to n and setting the derivative equal to zero gives the number of resonators which should be used to minimize the midband dissipation loss for the specified value of R. The result is

$$n = \frac{\log_e R}{2} = 1.15 \log_{10} R. \tag{20}$$

If instead of R being given, the attenuation is given as A decibels at a specified frequency, where the electrical length of each quarter-wave transformer section is θ_1, then[4] for large R

$$A = 10 \log_{10}\left[\frac{R}{4}(\cos\theta_1)^{2n}\right] \tag{21}$$

provided that R is large enough and θ_1 is far enough into the stop band. One then finds from (20) and (21) that

$$n = 0.115A + 0.7 - 23n \log_{10}(\cos\theta_1) \tag{22}$$

$$\approx 0.115A + 0.7 \tag{23}$$

when θ_1 is small. Thus for narrow-band filters only the specified attenuation (and not the specified frequency) determines n. For instance if $A=60$ db, $n \approx 7.6$, that is, 7 or 8 resonators should give minimum dissipation loss. This is in fair agreement with Cohn[10] where the minimum dissipation loss coccurs between 6 and 7 resonators when the 60-db bandwidth is specified. Eq. (23) has also been derived independently by Taub.[11] [Note added in proof: A similar formula has recently been given by Kogan.[12]]

[6] L. Young, "Prediction of absorption loss in multilayer interference filters," *J. Opt. Soc. Am.*, vol. 52, pp. 753–761; July, 1962.

[7] S. B. Cohn, "Dissipation loss in multi-coupled-resonator filters," Proc. IRE, vol. 47, pp. 1342–1348; August, 1959. See eq. (1).

[8] L. Young, "Q-factors of a transmission line cavity," IRE Trans. on Circuit Theory, vol. CT-4, pp. 3–5; March, 1957.

[9] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc., New York, N. Y., p. 220; 1945.

[10] Cohn, *op. cit.*, see Fig. 6.

[11] J. J. Taub, "Design of minimum loss bandpass filters," to be published in the Proc. IEEE.

[12] S. Kh. Kogan, "Efficient design of band filters with small dissipative losses," *Radio Engrg. and Electronic Phys.*, vol. 7, (English translation, Pergamon Press), pp. 1238–1244; August, 1962.

Conclusion

Group-delay and dissipation loss in stepped-impedance transmission-line filters has been considered. The formulas are novel in that they were derived by transmission-line methods and use only transmission-line parameters. The formulas reduce to known formulas for lumped-constant filters in the case of narrow-band selective filters, but hold more generally for stepped-impedance filters of any bandwidth and any selectivity.

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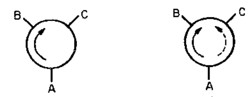
Reciprocal and Nonreciprocal Switches Utilizing Ferrite Junction Circulators*

The symmetrical ferrite junction circulator has assumed a position of prominence in recent years due to its small physical size and excellent electrical characteristics. The device was described theoretically by Auld,[1] and other investigators have contributed to the design of improved electrical characteristics.[2-4] Due to their low loss, these circulators have found wide use with parametric and tunnel diode amplifiers. Other applications include duplexers, multiplexers and load isolators.

As with all types of circulators, if the sense of the magnetic field is reversed, the direction of circulation reverses, making the design of modulators and switches possible. It is the purpose of this communication to emphasize these properties of junction circulators and discuss possible circuits for both reciprocal and nonreciprocal switching. Some of the experimental characteristics of these devices will also be presented.

The junction circulator is represented schematically as indicated in Fig. 1(a) where the arrow indicates the direction of circulation. The implication is that power entering at Port A leaves at Port B, or power entering at Port B leaves at Port C, etc. A switchable circulator is depicted in Fig. 1(b). The schematic has both a solid and dashed arrow representing the direction of circulation in the two states of the applied magnetic field. In one position, the direction of the circulation is ABC and in the switched position, represented by the dashed arrow, the direction of circulation is ACB. It is obvious that the circuit of Fig. 1(b) represents a single pole-double throw switch; that is, an input at Terminal A can be switched to Terminal B or Terminal C by alternating the direction of applied magnetic field. A photograph of such a switch designed for the S-band region is shown in Fig. 2.



(a) (b)

Fig. 1—(a) Circulator. (b) Switchable circulator.

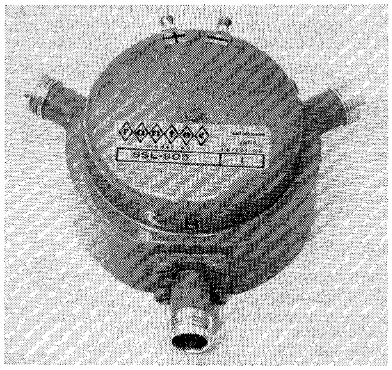


Fig. 2—Switchable circulator for the 2.0-kMc band. The unit performs the function of a nonreciprocal single pole-double throw switch.

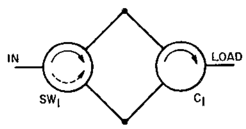


Fig. 3—Reactive, single pole-single throw reciprocal switch.

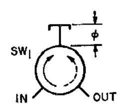


Fig. 4—Reactive balanced modulator or switch.

There are several properties of this type of junction switch that should be pointed out. First, the device is nonreciprocal; second the impedance is not constant during switching; and third, fast switching times are difficult to achieve.

The first property is apparent from a study of Fig. 1(b). When the switch is connected between A and B, B is connected to C instead of A and duplexing of the switched signal cannot be accomplished. Furthermore, the isolation between the two loads at B and C is determined, not only by the circulator, but by how well the loads are matched to the transmission line.

The second item can be understood by considering the nature of the circulator at zero field. In this condition the device is a *reciprocal* 3-port (circulation occurs in both directions with equal amplitude) and cannot be matched bilaterally from port to port. The VSWR can be as high as 2:1 during switching. If this condition is intolerable to the system, it can be overcome by the addition of an isolator at the input of the switch.

The difficulty in achieving fast switching times in these devices is caused by demagnetizing fields and eddy currents. The strong demagnetizing fields in the circulator are due to the disk-shaped ferrite geometries used. These fields oppose the applied field and more switching power is required. The demagnetizing field can be reduced by making the ferrite longer with respect to its diameter.

Eddy currents are present in the coil forms as well as in the ground planes and the center strip of the circulator transmission lines. These currents are difficult to reduce in practice. "Thin wall" techniques have been tried with some success; however, it is felt that 100 μsec seems a practical limit for these devices using reasonable switching power. One μsec switching does not appear practical at this stage of development.

In the discussion to follow, a number of different switching circuits using 3-port junction circulators will be presented. These circuits should cover most switching requirements, both reciprocal and nonreciprocal.

Single Pole-Single Throw (Reciprocal)

A single pole-single throw reciprocal switch can be obtained by combining the switchable circulator with a nonswitchable 3-port circulator. The arrangement utilized for this function is shown in Fig. 3. When the switchable circulator is in the state shown by the solid arrow, the input is connected to the load, and the load is similarly connected to the input. In the position indicated by the dashed arrow, power flows from the input around the loop containing C_1 and back out the input without being connected to the load. Similar power flow occurs looking into the load terminal. This circuit, therefore, is reactive in nature and the energy is either transmitted or reflected from the input in the two switched positions. This circuit is the basic building block of some of the switches that follow.

Balanced Switch (Reciprocal)

A balanced switch having single pole-single throw characteristics is shown in Fig. 4. In this device, one of the three ports is terminated in an adjustable length short circuit. In one direction of circulation, indicated by the solid arrow, the power at the input travels to the short circuit, is reflected and emerges from the output. In the other direction of circulation, indicated by the dashed arrow, the power at the input feeds directly to the output. Therefore, in *both* switched positions, the input is connected to the output. At zero field the direction of circulations are opposite and equal in magnitude. It is then possible to adjust the phase length (ϕ) of the short circuit so that at the output the two paths will be 180° out of phase and the energy will be reflected back to the input. When this balance is realized, the switch has total reflections at zero field and is transmitting in either direction of the applied field. This device, besides acting as a switch, will provide the function of balanced modulation if its solenoid is driven by a symmetrical periodic waveform. That is, the output will contain an upper and lower sideband spectrum with a suppressed carrier. For broad-band performance, ϕ should have the smallest possible value.

* Received February 19, 1963.

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[2] L. Davis, U. Milano, and J. Saunders, "A strip line L-band compact circulator," Proc. IRE (*Correspondence*), vol. 48, pp. 115–116; January, 1960.

[3] J. B. Thaxter and G. S. Heller, "Circulators at 70- and 140-kMc," Proc. IRE (*Correspondence*), vol. 48, pp. 110–111; January, 1960.

[4] J. Clark, "Miniaturized, temperature stable, coaxial Y-junction circulators," IRE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-9, pp. 267–269; May, 1961.